

**54. IWK**  
Internationales Wissenschaftliches Kolloquium  
International Scientific Colloquium



**Information Technology and Electrical  
Engineering - Devices and Systems, Materials  
and Technologies for the Future**



Faculty of Electrical Engineering and  
Information Technology

Startseite / Index:

<http://www.db-thueringen.de/servlets/DocumentServlet?id=14089>

## Impressum

Herausgeber: Der Rektor der Technischen Universität Ilmenau  
Univ.-Prof. Dr. rer. nat. habil. Dr. h. c. Prof. h. c.  
Peter Scharff

Redaktion: Referat Marketing  
Andrea Schneider

Fakultät für Elektrotechnik und Informationstechnik  
Univ.-Prof. Dr.-Ing. Frank Berger

Redaktionsschluss: 17. August 2009

Technische Realisierung (USB-Flash-Ausgabe):  
Institut für Medientechnik an der TU Ilmenau  
Dipl.-Ing. Christian Weigel  
Dipl.-Ing. Helge Drumm

Technische Realisierung (Online-Ausgabe):  
Universitätsbibliothek Ilmenau  
[ilmedia](#)  
Postfach 10 05 65  
98684 Ilmenau

Verlag:  Verlag ISLE, Betriebsstätte des ISLE e.V.  
Werner-von-Siemens-Str. 16  
98693 Ilmenau

© Technische Universität Ilmenau (Thür.) 2009

Diese Publikationen und alle in ihr enthaltenen Beiträge und Abbildungen sind urheberrechtlich geschützt.

ISBN (USB-Flash-Ausgabe): 978-3-938843-45-1  
ISBN (Druckausgabe der Kurzfassungen): 978-3-938843-44-4

Startseite / Index:  
<http://www.db-thueringen.de/servlets/DocumentServlet?id=14089>

# IMPROVING POWER SYSTEM STATE ESTIMATION WITH PMU

*Michał Powalko, Krzysztof Rudion*

Otto-von-Guericke University Magdeburg, Germany

## ABSTRACT

This paper deals with the state estimation (SE) of power systems (PS). Presently power systems need to be able to handle the mix of conventional energy with the energy coming from dispersed generation (DG) and renewable energy sources (RES). In order to ensure stable and proper operation of such an expanded system the monitoring and controlling solution should be supplied with an accurate measurement data. Modern measurement devices like a phasor measurement unit (PMU) can deliver both highly accurate as well as synchronous measurement values of the voltages and currents magnitudes, phase angles and frequency. The main aspect of the study presented in this contribution will consider development of the SE in such a way, that the synchronous measured phase angle can be implemented and used in an algorithm.

**Index Terms** – dispersed generation, phasor measurement unit, renewable energy sources, state estimation, system observability.

## 1. INTRODUCTION

The development of the energy sector and areas associated with it results in the improvement of existing and development of new technologies not only to produce but also to consume the energy. The amount of the new kind of generation units called dispersed generation is growing rapidly, which is an effect of support by the national governments and the European Union. This leads to minimizing the energy transport losses due to covering part of the energy consumption through the local DG. It has also an impact on the behavior of the whole power system, since the locally generated power can not always be fully consumed at the same location and needs to be transported to other parts of the power system utilizing the distribution and transmission system. These new challenges that have been introduced by putting a significant amount of DG into operation are making the issue of providing electricity to the consumer at a proper quality level increasingly complex. A new monitoring and controlling solution concept needs to be developed in order to be able to take full advantage of the usage of dispersed generation. The point of interest should especially be focused on the networks with a high penetration of renewable energy sources, where the stochastic character of this kind of energy generation needs to be taken into account, since the requirement about the

"renewable energy" share in the global energy market is growing and for some countries has reached a level, where already the distribution network operators can point out some remarkable situations.

## 2. SYSTEM OBSERVABILITY ISSUE

### 2.1. System Observability - General Information

The power system consists of three main element groups: power generation, power transmission and power consumption. Controlling the work of this structure requires being able to influence the behavior of each of those elements. In order to move the system from one operating point into another, the actual conditions, i.e. point of operation, need to be known. To describe the current state of the electrical power system the knowledge of complex voltages at all nodes is required.

In the real power systems the whole set of those measurements is not provided, however it is possible to calculate the missing data to some extent. This calculation can be done using the state estimation process. Performing the SE requires fulfilling the specific requirements, which can be verified by performing the system observability (SO) analysis. In general the results of the SO study (graphical or numerical methods are available) provide the information, if the available number of the measurements and their placement in the system make it possible to execute the state estimation.

Taking into account the fact that the structure of the network is currently changing due to increased operation of more DG units and new load/storage solutions (e.g. Car-2-Grid [1]), the statement of the partial observability or incomplete observability was introduced. This describes only the selected part of the system, where an adequate amount of properly placed measurement exists to perform the state estimation. This is a bridge solution, which makes it possible to estimate at least a part – an island, of the whole power system.

### 2.2. Phasor Measurement Units

Following the technical progress in the field of measurement technology, phasor measurement units can be introduced as a solution to improve the conventional way of dealing with the observability and controllability of the power system [2]. PMU are devices, which can advantageously use the global positioning system (GPS), in order to extract the Coordinated Universal Time (UTC) and date information– see Figure 1.



$$\begin{bmatrix} I_{ij} \\ I_{ji} \end{bmatrix} = \begin{bmatrix} y_{ii}^j & y_{ij} \\ y_{ji}^i & y_{jj} \end{bmatrix} \begin{bmatrix} U_i \\ U_j \end{bmatrix} = \mathbf{Y}_{i-j} \begin{bmatrix} U_i \\ U_j \end{bmatrix} \quad (3)$$

Inside the weight matrix  $\mathbf{R}$  the information about the variance of the measurement errors is saved – in a diagonal form:

$$\mathbf{R}^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{\sigma_m^2} \end{bmatrix} \quad (4)$$

For the non linear system the minimized objective function from equation (1) can be expressed through (5) and needs to be solved in an iterative way.

$$\mathbf{G}(\mathbf{x}^k) \Delta \mathbf{x}^{k+1} = \mathbf{H}^T(\mathbf{x}^k) \mathbf{R}^{-1} (\mathbf{z} - \mathbf{h}(\mathbf{x}^k)) \quad (5)$$

On the left side, the matrix  $\mathbf{G}(\mathbf{x})$  is known as a gain matrix, which is a product of the Jacobian matrix built from the measurement functions in  $\mathbf{h}(\mathbf{x})$  and a weight matrix  $\mathbf{R}$ :

$$\mathbf{G}(\mathbf{x}^k) = \mathbf{H}^T(\mathbf{x}^k) \mathbf{R}^{-1} \mathbf{H}(\mathbf{x}^k) \quad (6)$$

Defining the right side of the equation (5) using the new variable  $\mathbf{b}(\mathbf{x})$ :

$$\mathbf{b}(\mathbf{x}^k) = \mathbf{H}^T(\mathbf{x}^k) \mathbf{R}^{-1} (\mathbf{z} - \mathbf{h}(\mathbf{x}^k)) \quad (7)$$

The problem of the state estimation issue can be written in a compact form:

$$\mathbf{G}(\mathbf{x}^k) \Delta \mathbf{x}^{k+1} = \mathbf{b}(\mathbf{x}^k) \quad (8)$$

and needs to be solved iterative in order to obtain:

$$\Delta \mathbf{x}^{k+1} = \mathbf{x}^{k+1} - \mathbf{x}^k \quad (9)$$

Because the gain matrix has a sparse, symmetric form, is generally positive definite and might be not invertable, the Cholesky decomposition can be used [6]:

$$\mathbf{G}(\mathbf{x}^k) = \mathbf{L} \cdot \mathbf{L}^T \quad (10)$$

in order to create a lower triangular matrix  $\mathbf{L}$  and use the forward/backward substitution method, which can be performed with a command built in MATLAB®:

$$\Delta \mathbf{x}^{k+1} = \mathbf{L}/(\mathbf{L}/\mathbf{b}) \quad (11)$$

To provide satisfactory accuracy of the estimation process, the input data should be as accurate as possible. The statement "accurate data" considers not only the accuracy of the measurement, but also the proper information regarding actual network topology and parameter of the network elements.

The possible contribution of PMU to the SE process described above will be discussed in following chapters. Generally, phasor measurement units can be used in addition to conventional measurement devices to provide more accurate data but also to transform the existing critical measurements into the redundancy measurement. The last mentioned issue is very important from the "bad data detection" point of view, since it is not possible to recognize bad values coming from the critical measurement.

### 3.2. Synchronous Measurements in SE

The phasor measurement unit described in chapter 2.2 uses the GPS signal in order to extract from it the time information and use it as a global synchronization source. The benefit of this approach is that the delay between performing the measurement with different devices put into operation in the whole system can be drastically minimized. Considering the 50Hz power system and the GPS time accuracy of 1μs [4], the possible error connected with the measurement synchronization is equal to 0.018 deg. Such measurement of voltage and currents values at the same sampling time can be called synchronized and allows for getting time precise magnitude values as well as the phase angles, which is not possible without the use of the global synchronization source. The opportunity to implement these new parameters into the state estimation algorithm and improve the accuracy of the results exists.

### 3.3. Time Stamped Measurements

The time information that the PMU extracts from the GPS signals is, in addition to the synchronization issue, used to save the measurement results with the second of century (SOC) tag – according IEEE Standard [4]. This UTC-time and date information can in practice be used for pre-filtering of the measurement data coming into SE – it is possible to find out the measurements taken at an exact time. That can help eliminate the situation, where due to communication delays the measurement data coming from the system are retrieved at different times, which can have a huge impact on the results of the calculation because of the behavior of power systems - measurements taken even a few milliseconds after each other can differ significantly. Additionally, the time tag gives the opportunity to create a database containing information for the off-line analysis.

### 3.4. Synchronous Measured Phase Angles

Generally the advantage of having knowledge about the proper value of the synchronized measured phase angles can be used for merging observable islands of an unobservable system. Assuming that observable areas are connected with unobservable branches, putting into operation PMU's in each of the islands makes it possible to estimate the state of the whole system (common reference phase value).

Typically in the estimation process one of the nodes in the network is chosen to be a reference node and the assumption that its phase angle is equal to zero is made. This is proper proceeding since there is no phase angle measurement in the network. If at the reference bus the PMU measurement is missing, but they are installed at some nodes in the network, the measured phase angles cannot be used in the estimation process due to an inconsistent relationship to the reference bus.

On the other hand, the error at phase angle measurement at the reference bus can not be detected and can effect other “proper” measurements. Since PMU can synchronously obtain the complex values of the voltage at the bus and the currents at the lines, having the knowledge about the parameters of power system elements represented with  $\pi$  model (Figure 3), equation (3) can be used to represent the estimation problem linearly:

$$\mathbf{z} = \mathbf{B}\mathbf{x} \quad (12)$$

where:

- $\mathbf{z}$  - matrix containing the PMU measurements (complex voltages and currents),
- $\mathbf{B}$  - matrix converting the measurements into states variables,
- $\mathbf{x}$  - state variable (complex voltages).

The elements of the  $\mathbf{B}$  matrix contain only the admittance values, so this makes it a linear formulation [3], which does not need to be solved in an iterative way. The WLS solution of this problem is given by:

$$\mathbf{x} = (\mathbf{B}^T \mathbf{R}^{-1} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{R}^{-1} \mathbf{z} = \mathbf{M}\mathbf{z} \quad (13)$$

Since the topology of the network does not change, the elements of the  $\mathbf{B}$  matrix stay also constant. Additionally, knowing the elements of the weight matrix  $\mathbf{R}$ , the new matrix  $\mathbf{M}$  can be calculated once at the beginning and stored for further use of estimation of the state variables. After performing the linear estimation the calculated values can be used as an input for conventional estimation for those parts of the network, where linear estimation was not possible.

#### 4. APPROACH AND STUDY CASES

The approach of implementing measurements coming from the PMU into the state estimation will be presented in this section. It will consider taking into account the synchronous measured phase angle during the estimation process. This issue will be discussed as the basis of the performed calculation on the example network shown in Figure 4. For solving the state estimation problem the WLS algorithm was used. Three scenarios were analyzed covering different aspects of incorporating the PMU measurements:

- Scenario 1 – standard SE calculation,
- Scenario 2 – standard SE with added synchronous measured phase angle into iterative algorithm,
- Scenario 3 – linear SE using PMU measurements (complex currents and voltages), additionally standard SE to obtain remaining state values.

The results of each case study - the residual of measurements (difference between measured values and the values calculated from measurement functions – Eq. (14)) and estimated state variables will be summarized in order to compare the results.

$$\text{Res}(\mathbf{x}) = \mathbf{z} - \mathbf{h}(\mathbf{x}) \quad (14)$$

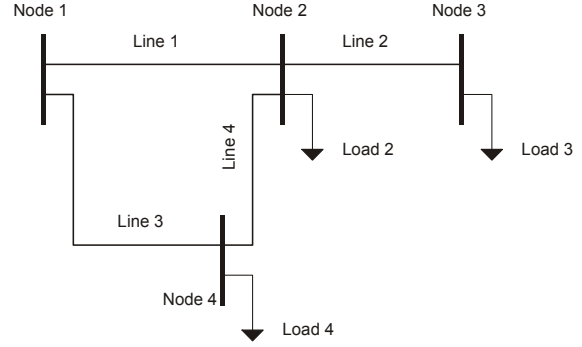


Figure 4 Benchmark network

The state variables that need to be estimated according to the network structure are shown in equation (15). Corresponding to the accuracy classes of the measurement devices the proper values of the weight matrix  $\mathbf{R}$  were calculated.

$$\mathbf{x}^T = [\varphi_2 \ \varphi_3 \ \varphi_4 \ V_1 \ V_2 \ V_3 \ V_4] \quad (15)$$

where:  $V_1 - V_4$  are the voltage magnitudes in the corresponding nodes and  $\varphi_2 - \varphi_4$  are the voltage phase angles at the corresponding nodes. Since the node number one was selected to be a reference, the phase angle value of its voltage does not appear in (15).

The outcomes carried out from the MATLAB® simulation (direct measurements on each node – not containing “measurement error”) were used as a reference. The first state estimation performed excludes input of synchronous measurements – the standard estimation process. The second scenario deals with the incorporation of measured voltage magnitude and phase angle into the estimation process. Those variables will be placed in measurement vector  $\mathbf{z}$  and the corresponding equations (16) and (17) will appear in the matrix  $\mathbf{h}(\mathbf{x})$  with measurement functions, where  $e_i$  is a real and  $f_i$  an imaginary part of the voltage.

$$u_i = \sqrt{(e_i)^2 + (f_i)^2} \quad (16)$$

$$\varphi_i' = \arctg(f_i/e_i) \quad (17)$$

In order to use the measured phase angle (issue described in chapter 3.4), the assumption was made that the value at the reference bus is known and was used to calculate the difference  $\Delta\varphi$  between the true reference value  $\varphi_1$  and a new set to zero  $\varphi_1'$  (common approach of setting the point of the origin for the system to zero) – equations (18) and (19).

$$\Delta\varphi = \varphi_1 - \varphi_1' \quad (18)$$

$$\varphi_1' = 0 \quad (19)$$

Knowing the  $\Delta\varphi$  value, it was then possible to use the measured phase angle  $\varphi_i$  oriented to a new common reference  $\varphi_1'$ .

$$\varphi_i' = \varphi_i - \Delta\varphi \quad (20)$$

Although this case implements a new parameter coming from the PMU it still requires an iterative solution to obtain the state variables (15).

The third simulated case takes full advantage of performing synchronous measurement of complex

voltages and currents (as described in chapter 3.4.) in order to create a linear description of the dependencies between the measurements ( $\underline{I}_{41}$ ,  $\underline{I}_{42}$ ,  $\underline{U}_4$ ) and state variables to solve a linear equation problem instead of performing the solution of non-linear estimation. Additionally, to obtain the absent voltage  $\underline{U}_3$  the standard SE was performed.

## 5. SIMULATION AND RESULTS

The simulations were performed at the benchmark network created in MATLAB®/Simulink® software with usage of the SimPowerSystems Toolbox [9]. The summary of the elements used in the network and their parameters can be found in Table 1 and Table 2. The nominal voltage of the system was 0.4kV at 50Hz frequency.

Table 1 Parameters of the lines

Node		R [Ω/km]	L [mH/km]	C [μF/km]	Length [km]
A	B				
1	2	0,01273	0,9337	0,01274	3
2	3				2
1	4				2
4	2				1

The values of the load powers were set at the beginning of the simulation and were assumed to be constant – the block “Three-Phase Series RLC Branch” was used to model those elements.

Table 2 Parameters of loads

Node	Active power [kW]	Reactive power [kVar]
2	5	1
3	1	0
4	2	0

The list of performed measurements taken as an input to state estimation, the assumed accuracy classes of the measurement devices and calculated standard deviation values used for calculation of the weight matrix are summarized in Table 3. The results of the estimation within the iterative way – for two first scenarios, are shown in Table 4 and Table 5.

Table 3 Measurements – place, accuracy classes and standard deviations

Available measurements			Accuracy [%]	Standard deviation
Scen. 1	Scen. 2	Scen. 3		
$P_{12}$	$P_{12}$	---	1,00	3,33E-01
$P_{23}$	$P_{23}$	$P_{23}$		
$P_{41}$	$P_{41}$	---		
$Q_{12}$	$Q_{12}$	---		
$Q_{23}$	$Q_{23}$	$Q_{23}$		
$Q_{41}$	$Q_{41}$	---		
$U_2$	$U_2$	---		
$U_4$	---	---	0,20	1,33E-02
---	$\underline{U}_4$	$\underline{U}_4$		
---	---	$\underline{I}_{42}$		
---	---	$\underline{I}_{41}$		

Table 4 Results – Scenario 1

Measurement	Measured value	Estimated value	Residual
$P_{12}$ [W]	3496,6537	3496,6536	1,168E-04
$P_{23}$ [W]	933,5927	933,5927	1,409E-08
$P_{41}$ [W]	-4131,8988	-4131,8992	4,008E-04
$Q_{12}$ [Var]	533,8906	533,8817	8,879E-03
$Q_{23}$ [Var]	2,2719	2,2719	2,196E-09
$Q_{41}$ [Var]	-479,7743	-479,7803	5,958E-03
$U_2$ [V]	226,5261	224,2529	2,2733
$U_4$ [V]	222,2887	224,5617	-2,2730

For the state estimation calculations the convergence requirement shown in (21) was taken into account.

$$\Delta x^{k+1} < 1E - 6 \quad (21)$$

For both of those cases 5 iterations were needed to meet the requirement described above in (21). The algorithm provided a satisfactory solution for most of the measurements – the values of the residual are marginal, except the voltages.

Table 5 Results – Scenario 2

Measurement	Measured value	Estimated value	Residual
$P_{12}$ [W]	3496,6537	3496,6536	1,137E-04
$P_{23}$ [W]	933,5927	933,5927	4,036E-11
$P_{41}$ [W]	-4131,8988	-4131,8992	3,888E-04
$Q_{12}$ [Var]	533,8906	533,8819	8,639E-03
$Q_{23}$ [Var]	2,2719	2,2719	3,970E-12
$Q_{41}$ [Var]	-479,7743	-479,7801	5,797E-03
$U_2$ [V]	226,5261	224,3137	2,2124
$\phi_4$ [°]	-0,9021	-1,3639	0,4618
$U_4$ [V]	224,6340	224,6225	0,0115

The second simulation shows, that despite taking into account the phase angle during the estimation process, the values of the residuals have only been improved partially. Although the voltage measurement at node 4 was performed with better accuracy than in the first scenario, the higher weight value makes it impossible for the algorithm to find a better solution - see Table 7.

In the last simulation – Scenario 3, the linear model of the system was created and solved using the measurement performed through PMU installed at node 4. The result of the calculated state variable (voltage  $\underline{U}_2$ ) was further used within the power flows ( $P_{23}$  and  $Q_{23}$ ) measurement in conventional state estimation to obtain the complex voltage value  $\underline{U}_3$ . Table 6 shows the residual of the iterative solution of the voltage mentioned above. The comparison of state variables estimated in Scenario 1, 2 and 3 is summarized in Table 7.

Table 6 Results – Scenario 3

Measurement	Measured value	Estimated value	Residual
$P_{23}$ [W]	938,3078	938,3078	6,8212E-13
$Q_{23}$ [Var]	2,2606	2,2606	-5,9668E-12
$U_2$ [V]	224,0592	224,0592	5,6843E-14

Table 7 State variable – estimation results

State variable	Real value	Estimation error: Angle [ $\Delta^\circ$ ], Voltage [%]		
		Scenario 1	Scenario 2	Scenario 3
$\varphi_1 [^\circ]$	-4,3536	---	---	0,0057
$\varphi_2 [^\circ]$	-5,4981	0,5874	0,5865	0,0053
$\varphi_3 [^\circ]$	-5,7082	1,8340	1,8329	0,1088
$\varphi_4 [^\circ]$	-5,2557	1,8171	1,8164	0,0053
$U_1 [V]$	225,1381	-0,1550	-0,1419	0,0046
$U_2 [V]$	224,2833	0,0136	-0,0136	-0,0003
$U_3 [V]$	224,2466	0,0205	-0,0167	0,0067
$U_4 [V]$	224,5340	-0,0123	-0,0394	0,0003

For the scenario without phase angle measurement at reference bus (node 1), the proper values of phase angles were calculated according to Eq. (20). The same procedure was executed for a complex voltage at node 3 in the Scenario 3. It can be noticed, that between the basis simulation - Scenario 1 and the one with incorporated phase angle into non-linear iterative estimation - Scenario 2, only partial improvements were achieved. The last case – linear estimation in Scenario 3 - was the only case in which a significant enhancement of accuracy was obtained.

## 6. SUMMARY

This paper deals with the state estimation procedure and incorporation of additional measurement parameters obtained with a new technique provided by the phasor measurement devices. The conventional algorithm of state estimation with an iterative solution approach was presented and additionally, the unique features of PMU's, which can contribute to the extension of the estimation algorithm, were pointed out. The global synchronization source makes it possible to perform the synchronized measurement of complex voltage and current values, which can be used as an additional input to SE. On the example test network different ways of implementing those measurements were analyzed. It was noticed, that expanding the measurement vector of measured phase angle for the iterative solution of the estimation problem can partially improve the accuracy. However, the most efficient way to take full advantage of the synchronous measurement is to perform the linear estimation of state variables, which not only makes the SE process more accurate, but also simplifies the computation issues. Thus, because matrix  $\mathbf{B}$  converting the synchronous measurements into state variable is constant (when the structure of the system does not change), the variables of  $\mathbf{B}$  do not need to be calculated again when a new measurement data set needs to be used in SE (as in the gain matrix  $\mathbf{G}(\mathbf{x})$  for the non linear estimation). This makes the calculation of a huge power system more efficient.

It can be also noticed that in comparison to the iterative way of solving the estimation problem, the linear representation of the relationships between synchronized measurements (currents and voltages) and state variables - complex voltage values, depending on the network structure, can reduce the amount measurements required to perform an estimation.

## 7. REFERENCES

- [1] P. Lombardi, P. Vasquez, Z. A. Styczynski; "Plug-in Electric Vehicles as storage devices within an Autonomous Power System. Optimization issue", PowerTech 2009, Bucharest, Romania, 28 June - 2 July 2009.
- [2] Z. A. Styczynski, K. Rudion, P. B. Eriksen, A. G. Orths, T. Schäfer, W. Sattinger, L. Rouco, A. Phadke; "Operation and Control Strategies for Networks with a High Degree of Renewable Generation", Survey Paper at PSCC 2008 Conference, Glasgow, Scotland, 14-18 July 2008.
- [3] A. G. Phadke, J. S. Thorp; "Synchronized Phasor Measurements and Their Applications", Springer Science+Business Media, LCC, 2008.
- [4] IEEE Standard for Synchrophasors for Power System (IEEE C37.118™ -2005, revision of IEEE Std 1344™ -1995), Approved 1 February 2006 American Nation Standards Institute Approved 21 October IEEE-SA Standards Board.
- [5] ABB Power Technologies AB; "Technical Reference Manual RES 521\*1.0", 2008.
- [6] A. Abur, A. G. Exposito; "Power System State Estimation – Theory and Implementation", CRC Press, 2004.
- [7] P. Kundur; "Power System Stability and Control", McGraw-Hill, Inc, 1994.
- [8] M. Powalko, K. Rudion, P. Komarnicki, J. Blumschein; "Observability of the Distribution System", The 20<sup>th</sup> International Conference and Exhibition on Electricity Distribution - CIRED 2009, Prague, Czech Republic, June 2009.
- [9] The MathWorks - MATLAB and Simulink for Technical Computing, version R2007a, available at: [www.mathworks.com](http://www.mathworks.com).